# MATHEMATICAL MODELLING OF THE SEISMIC RESPONSE OF A ONE STORY STEEL FRAME WITH INFILLED PARTITIONS

# B.S. Yanev and H.D. McNiven

# SYNOPSIS

The response of a one story steel frame to earthquake excitations is discussed. The contribution of infilled partitions to the structural behavior is examined. Results of experiments performed at the Earthquake Engineering Research Center, University of California, Berkeley with one storv steel frames are described. System identification techniques are applied to the experimental data in order to determine physically meaningful parameters representing the behavior of the structure. The effect of highly nonlinear material behavior on such parameters and on the overall structural response is demonstrated.

#### RESUME

Dans cet article on discute de la réponse aux séismes d'un cadre en acier d'un étage. On étudie en particulier l'effet des éléments de remplissage sur le comportement structural. On décrit les résultats d'essais sur ce type de cadres, réalisés au centre de recherches sur les séismes (University of California, Berkeley). On applique aux résultats expérimentaux des techniques d'identification permettant de déterminer les paramètres significatifs pour le comportement de la structure. On démontre l'effet du comportement non linéaire des matériaux sur ces paramètres et sur la réponse globale de la structure.

Bojidar S. Yanev obtained his E.Sc.D. from Columbia University in New York in 1976. He is currently Assistant Research Engineer at the Earthquake Engineering Research Center, University of California, Berkeley.

Hugh D. McNiven obtained his Ph.D. from Columbia University in New York in 1958. He is currently Professor of Engineering Science at the University of California in Berkeley and has been a faculty member of the Earthquake Engineering Research Center since 1972.

# INTRODUCTION

One of the major concerns of the structural engineer today is to develop the ability to predict the responses, both linear and nonlinear of buildings to seismic forces. While it is difficult to assess the state of the art it is fair to say that, although a great deal is known of linear behavior of buildings, we are at the early stages in understanding much of the linear response and most of the nonlinear one. ----

When, for example, we discuss the probable response of a building, we usually refer to the response of the building frame. The fact that this response does not account for the influence of infill walls and partitions is a source of growing dissatisfaction. The extent of such influences and the means for their full utilization are not determined.

Qualitatively, one expects infill partitions to reduce frame displacements and to absorb energy but there is little understanding of the magnitude of these effects. The considerations whether or not walls and partitions should be accounted for in calculating the seismic response of a building are inconclusive.

The study reported here is a first step in assessing as quantitatively as possible the influence of infill partitions on the response of a one story steel frame subjected to earthquake motions in the plane of the partitions. The study consists of two parts, experimental and analytical.

The experimental program is made possible by the earthquake simulator of the Earthquake Engineering Research Center at the University of California in Berkeley. Initially, experiments are performed with a bare single story steel frame (Fig. 1-b), reminiscent of the frame (Fig. 1-a) which has been tested and modelled mathematically by Matzen and McNivem (1). After the response of the bare frame is recorded a number of different types of partitions are built into the bays of the structure as shown on Fig. 1-b. The resulting structure is subjected to similar earthquake motions.

Particularly helpful in gaining a quantitative insight into the

influence of the partitions is the formulation of mathematical models. The method has been used successfully in the past. Kaya and McNiven (2) for example have been able to gain an understanding of the behavior of a three story steel frame by constructing mathematical models, both linear and nonlinear, predicting almost exactly its response to an array of seismic forcing functions. In both studies (1) and (2) the techniques of system identification are used for constructing the models.

System identification is used once again in the present study. The paper offers a brief discussion of the method. System identification allows for considerable flexibilty and a variety of approaches. This paper only deals with some of them, particularly suited to the purposes of the problem.

A description of the experiments follows. Experimental results are displayed and compared to analytical ones. Conclusions are formed concerning the influence of infilled partitions on the seismic response of the frame and the applicability of system identification techniques to the problem of predicting structural behavior during earthquakes. An attempt is made to determine physically meaningful parameters of structural response and to investigate their behavior as the properties of the infill partitions change due to the earthquake motion.

# EQUATION OF MOTION

The frames of Fig. 1 are considered as one degree of freedom systems. The classical equation of motion for such systems is:

$$i\ddot{x} + C\dot{x} + P(x) = -M\ddot{x}_{a}$$
 (1)

where x - displacement of the concentrated mass of the system relative to the ground,

x - relative velocity,

x - relative acceleration,

 $\ddot{x}_{g}$  - ground acceleration,

M - mass of the system,

C - viscous damping,

P(x) - load - displacement relationship.

The choice of P(x) is a major step in the modelling procedure. In the simplest case of linear response:

$$P(\mathbf{x}) = \mathbf{K} \mathbf{x} \tag{2}$$

where K - stiffness of the structure.

Nonlinear structural behavior has been represented by Ramberg -Osgood (Fig. 2-a), bilinear (Fig. 2-b) and other models. In (1) it is concluded that the Ramberg - Osgood model is effective in duplicating the actual accelerations of the structure (Fig. 1-a) and provides a realistic value for the structural stiffness. It is noted that the yield of the frame introduces a discrepancy between the actual and the simulated displacements. Apart from that, displacements are accurately simulated. The viscous damping heavily depends on the time span of the considered experimental data.

In (2) the linear response of a three story frame is matched with excellent accuracy both in displacement and acceleration. In dealing with the problem of the 'plastic slip' in the steel structure at yield point Kaya suggests a 'tri-linear' model allowing for an additional adjustment of the post - yield stiffness.

Both results seem to indicate that inspite of their differences, both Ramberg - Osgood and bilinear models lead to a satisfactory simulation of the acceleration and displacement time histories recorded during earthquake excitations. Both models obtain similar and consistently realistic values for the structural stiffness in the linear range. The discrepancy in displacements after major yielding presents a problem to both models as does the erratic behavior of the viscous damping.

In (3) it is reported that the viscous damping of a steel structure is amplitude dependent and can increase by a factor of nearly 10 as the 'sensitivity limit' of the structure is exceeded. It is also noted that the energy dissipation through structural joints and through contact with the ground makes it difficult to separate the viscous damping from other energy absorbing factors.

These considerations suggest that the choice of P(x) is not solely responsible for the discrepancies in the representation of post - yield structural displacements. Assigning a constant value to represent the viscous damping before, during and after yield emerges as excessively simplistic. In the presence of infilled partitions whose contribution to the overall structural behavior is less known a similar viewpoint is taken of the structural stiffness. If partitions influence the stiffness of the frame, any model of the structural behavior should be able to represent the changes in the stiffness, corresponding to the deterioration of the partitions.

Finally, the nature of the earthquake excitations must be taken into account. Earthquake motions are random ones. The acceleration peaks causing nonlinear effects in the structure are usually separated by time periods during which the structure responds linearly or may even come to a standstill.

An attempt is made to develop an approach taking the above into account. The following development is not confined to any particular model. Because of its relative simplicity, a bilinear model (Fig. 2-b) is assumed for the purposes of the present discussion. As stipulated by the bilinear model, the linear response of the structure is characterized by a stiffness  $K_1$  and a viscous damping  $C_1$ . A set of new values  $K_2$  and  $C_2$  represents the post - yield behavior. After an inelastic displacement (up to point A on Fig. 2-b) a structure may continue to oscillate linearly. Should no subsequent acceleration peak exceed the yield load, such a structure would come to rest with a permanent eccentricity equal to  $\bar{x}$ . If point A denotes a reversal in the structural motion, beyond it the neutral position of the structure is  $\bar{\mathbf{x}}$ . Nontheless a solution of Eq. (1) over the entire time span of an earthquake implies initial conditions:

$$\mathbf{x}_0 = 0$$
;  $\dot{\mathbf{x}}_0 = 0$ ;  $\ddot{\mathbf{x}}_0 = 0$  (3)

An alternative is to consider each cycle of structural motion individually. At each point of motion reversal a check is made for the value of  $\bar{x}$ .

Equation (1) assumes the form:

$$M\ddot{x} + C\dot{x} + P(x - \bar{x}) = -M\ddot{x}_{z}$$
(4)

On the basis of the bilinear model of Fig. 2-b,  $\bar{x}$  is readily determined. For the half cycle originating at point A with a negative velocity:

$$\vec{k} = x_A + (\vec{x}_{gA} + \vec{x}_A) / \kappa_1$$
 (5)

Eq. (4) can be applied to spans of motion restricted by points of reversal. Different assumptions can be made for the behavior of C and P(x). A restriction is imposed by the length of the cycle which has to be sufficient for the implementation of a numerical procedure.

### NUMERICAL PROCEDURE

The gist of the identification problem is to select a model form representing the system behavior, an error (cost) function reflecting the performance of the model and a numerical procedure optimizing the parameters of the model. For a general description of identification techniques applicable to problems of structural dynamics the reader is referred to (4). The choice of the numerical procedures for error minimization and the solution of the differential equation are not limited to those used in obtaining the present results. Consequently the numerical aspects of the problem are not emphasized.

The following error functions are considered:

$$J_{1}(\bar{a},T) = \int_{T_{1}}^{T_{2}} (\ddot{y}(\bar{a},t) - \ddot{x}(t))^{2} dt \qquad (a)$$

$$\Psi_2(\bar{a},T) = \int_{1}^{T^2} (\dot{y}(\bar{a},t) - \dot{x}(t))^2 dt$$
 (b)

$$J_{3}(\bar{a},T) = \int_{T_{1}}^{T_{2}} (y(\bar{a},t) - x(t))^{2} dt \qquad (c)$$

$$J_4(\bar{a},T) = J_1(\bar{a},T) + b J_3(\bar{a},T)$$
 (d)

where a is the parameter vector, b is a weighting factor.

x and x are obtained from experiment.

 $\dot{\mathbf{x}}$  can be obtained by one of the various numerical procedures. (It was noted that the choice of the procedure does not affect  $\dot{\mathbf{x}}$  significantly.)

 $T_1$  and  $T_2$  define the beginning and the end of the time span over which the structural motion is considered. (The present results are obtained primarily for half and full cycles of motion.)

Two different methods are employed in obtaining v, v and v.

A. An initial assumption is made for the values of the model parameters. In the presently adopted bilinear model this implies two stiffness values  $K_1$  and  $K_2$  and a value of the vield load  $P_y$  denoting the change from  $K_1$  to  $K_2$ . Results have been compared for the case of the damping changing together with the stiffness (i.e.  $C_1$  and  $C_2$ ) and for a constant damping over half and full cycles. For these parameters and a given forcing function  $\ddot{x}_g$  the equation of motion is solved over the specified time span. Various techniques may be used for the numerical solution of Eq. 4. Newmark's linear acceleration method yields satisfactory results.

The error is evaluated according to Eq. 6-a, c or d. A Gauss -Newton procedure is used for the error minimization. A new set of parameters is selected and the procedure is repeated until a prescribed error tolerance is satisfied.

B. At each time step (i) within the time interval Eq. 4 is solved in one of the following forms:

$$\ddot{v}_{i} = -\ddot{x}_{gi} - (C_{i}\dot{x}_{i} + K_{i}(x_{i} - \bar{x}))/M$$
 (a)

$$\dot{y}_{i} = -(M(\ddot{x}_{ei} + \ddot{x}_{i}) + K_{1}(x_{i} - \bar{x}))/C_{1}$$
 (b) (7)

$$y_{i} = -(M(\dot{x}_{i} + \ddot{x}_{i}) + C_{1}\dot{x}_{i})/K_{1} + \ddot{x}$$
 (c)

for  $T_1 \neq T \neq T_n$  and

$$\ddot{\mathbf{y}}_{i} = -\ddot{\mathbf{x}}_{gi} - (C_{2}\dot{\mathbf{x}}_{i} + K_{1}(\mathbf{x}_{y} - \bar{\mathbf{x}}) + K_{2}(\mathbf{x}_{i} - \mathbf{x}_{y}))/M \quad (a)$$
  
$$\dot{\mathbf{y}}_{i} = -(M(\ddot{\mathbf{x}}_{gi} + \ddot{\mathbf{x}}_{i}) + K_{1}(\mathbf{x}_{y} - \bar{\mathbf{x}}) + K_{2}(\mathbf{x}_{i} - \mathbf{x}_{y}))/C_{2}(b) \quad (8)$$
  
$$\mathbf{y}_{i} = -(M(\ddot{\mathbf{x}}_{gi} + \ddot{\mathbf{x}}_{i}) + C_{2}\dot{\mathbf{x}}_{i} + K_{1}(\mathbf{x}_{y} - \bar{\mathbf{x}}))/K_{2} + \mathbf{x}_{y} \quad (c)$$

for  $T_y < T < T_2$ , where T denotes the time step at which the parameters are changed and  $x_y$  is the corresponding displacement.

The optimization of the parameters is carried out as in A. Error functions 6-a, b and c are applied to Eqs. 7 and 8-a, b and c respectivelv.

The second procedure which can be referred to as direct is considerably faster than the first one. It is particularly useful for a preliminary assessment of the structural response. A disadvantage of this procedure is the inability to treat the yield load as another parameter subject to identification. The change of the parameters has to correspond to an exact time step. Results have been compared after choosing different time steps for the change of the parameters. Procedure A eliminates this necessity. It can treat the yield load as an identifiable parameter. Thus if nonlinear behavior is not observed during a given cycle procedure A indicates that  $P_y$  has not been reached and K<sub>2</sub> has not been used while procedure B obtains K<sub>1</sub> = K<sub>2</sub>.

Both methods have been applied to experimental data. The results of simulating displacement and acceleration time histories obtained from experiments with the frames of Fig. 1-a and b are shown on Figs. 3 and 4 respectively. Solid lines indicate the simulated response, dotted lines denote the actual one.

Since this simulation is obtained by considering relatively short time spans, it is necessary to examine the results in view of the parameters determined in the process. It is observed that the values of the structural stiffness are consistent from cycle to cycle and in good agreement with results of straightforward structural analysis.

The significance of the displacement  $\bar{x}$  becomes apparent when the values of the damping are examined. These values are still apt to change within the range suggested in (3) as the stiffness changes. During subsequent cycles of linear response however, their behavior becomes much more regular.

Error functions  $J_1$ ,  $J_3$  and  $J_4$  and Eqs. 7 and 8-a and c yield similar results and can be relied upon for obtaining  $K_1$  and  $K_2$  even if  $\bar{x}$  is not introduced. Without  $\bar{x}$  however, error function  $J_2$  and Eqs.7 and 8-b can not be used. (Velocities  $\dot{x}$  are not available from experiment. They have been introduced here primarily to demonstrate the sensitivity of the viscous damping to nonlinear displacements and changes in the structural stiffness.)

As demonstrated on Fig. 3 and 4, the displacement simulation does not suffer from the effects of structural yield if  $\bar{x}$  is taken into account. This applies to both structures of Fig. 1 regardless of their significant differences. Certain details involved in the design of these frames are described along with the experiments performed with them.

### EXPERIMENTAL RESULTS

The experiments with the structures shown on Fig. 1 were performed on the shaking table of the Earthquake Engineering Research Center, University of California, Berkeley. The testing facility is described in detail in (5). It can simulate earthquake excitations of a prescribed form in three general directions. The table displacement can exceed 5". Acceleration reaches 1. G.

The structure of Fig. 1-a was tested as part of the research reported in (1). The frame of Fig. 1-b was designed and tested for the purposes of this study. It is less flexible. The columns are positioned so that rotation occurs around the major axis of the cross section. End connections are rigid. Braces prevent sidesway and twist.

The frame was subjected to excitations in the direction indicated on Fig. 1. The El Centro, Taft and Pacoima accelerograms were used. The magnitude of the impulse was varied and in many cases exceeded the actual one. The base and top acceleration were recorded at a scanning rate of 100 Hz. Similar time histories were obtained for the base and top displacements. During experiments designed to produce yield in the columns the strain in the areas of maximum deflection was also recorded.

The choice of the partitions and the design of the frame was motivated primarily by the desire to obtain realistic results, helpful to the designing engineer. The combination of a steel frame and infill panels, while not frequently encountered in construction proved adequate for the overall size of the experimental structure. Cross section type S 3 X 7.5 was used for the frame columns.

- The infill partitions were of the following types:
- (a) unreinforced common bond ungrouted masonry with unit size 3.60"/7.60"/3.80"
- (b) unreinforced common bond ungrouted masonry with unit size 1.80"/3.80"/1.00" (Miniature Giant, product of Clayburn Ind. Ltd., Bellevue, Wash.)
- (c) prefabricated panels (Cement Composite, product of Finestone Co., Detroit, Mich.)
- (d) Wood stud partitions

The size of the partitions was approximately 5 / 8.5 ft. The mass at the top of the structure was varied according to the stiffness of the partitions.

The forming and development of cracks was closely observed and in certain cases filmed. Of major interest was the behavior of the structural stiffness corresponding to the various stages of deterioration of the partitions. In this respect panel (a) proved an exceptional case. Its stiffness by far exceeded that of the frame and the experiment demonstrated the performance of the panel rather than that of the entire structure.

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In the rest of the experiments the stiffness of the frame and the partitions related in a way allowing for the development of nonlinear effects in both. The panels were tested with and without a gap between their three sides and the frame. Similar structures were alternatively subjected to gradually increasing and to major earthquake motions. Precautions were taken against fallout of the panels.

For the most part the results obtained for the overall structural response of panels (b) through (d) **are** comparable. The structural stiffness is increased by the presence of the partitions only until cracks form between the frame and the panels. Such cracks are the first to form and occur at an early stage of the experiment. The unreinforced masonry also develops cracks parallel to the horizontal edges of the partition (Fig. 5-b). Beyond this point the structural stiffness remains essentially the same as the stiffness of the bare frame.

The range of the linear response of the structure however, is substantially increased by the partitions. For the masonry wall (b) shown on Fig. 5 this effect is described in some detail.

The structure was subjected to the Pacoima earthquake motion with a maximum ground acceleration of approximately l. G. The crack pattern at the beginning and after the completion of the experiment is shown

on Fig. 5-b and c.

Fig. 6 and 7 are obtained directly from the experimental data by plotting the structural acceleration multiplied by the mass versus the structural displacement. There appears to be one major disturbance in the acceleration pattern which corresponds to a separation of the panel from the frame. For the remaining part the relationship between displacements and **pseudo-forces** is close to linear. A permanent displacement is introduced during the excitation. Beyond that point the stiffness is slightly reduced (and corresponds to the stiffness of the bare frame). The peak load values substantially exceed the yield load of the bare steel frame.

Fig. 8 represents a relationship between the strain at the end of a column and the overall structural displacement. In the case of the statically determinate frame of Fig. 1-a such a relationship would be expected to remain linear. Nonlinearity would be reflected in the load - displacement relationship.

In the present case however, the nonlinearity between the strain and the corresponding displacement is evident. The displacement varies proportionally to the load while the steel column is yielding.

The same conclusion is reached through the identification process which has resulted in the simulated acceleration and displacement time histories on Fig. 4. The stiffness values of the model indicate a linear response. The yield of the steel frame and the corresponding cracking of the wall (the acceleration irregularity of Fig. 6) are reflected by a sharp increase of the damping and a slight reduction of the stiffness for that particular cycle.

The experiment with partition (b) proceeded to the full destruction of the masonry which was brittle. Partitions (c) and (d) were extremely flexible. They continued to excercise the effect described above throughout tests of comparable magnitude. The observations made during these tests and the results of the system identification were in good agreement and pointed to several general conclusions.

### CONCLUSIONS

Among the conclusions reached during the reported work are the following:

A. The contribution of infill partitions to the behavior of a single story frame is demonstrated in the following aspects:

The overall stiffness of the structure is increased only until the partitions separate from the frame. The separation occurs in the form of cracks along the partition edges at an early stage of the experiment. Beyond that point the response of the structure remains roughly linear with a stiffness approximately equal to that of the bare frame. An eccentricity may be introduced in the structure during this process. The corresponding cycle or cycles of motion indicate disturbances in the acceleration.

The range of the linear response of the frame increases considerably in the presence of partitions. Modelling considerations are based on the knowledge of the material properties of the structure. However, it is also necessary to take into account the degree of structural redundancy. Internally, the redundancy of the frame is increased by the infill partitions. As a consequence, the effect of the nonlinear behavior of both frame and partitions on the overall structural response to earthquake motions is reduced.

B. System identification techniques are applicable to individual cycles of structural motion. Such an approach reflects the changing nature of the structural stiffness and damping without requiring an early commitment to an elaborate model. It is justified in cases when the duration of the cycles provides a number of points on the time history sufficient for the implementation of a numerical procedure.

Application of the classical one degree of freedom equation of motion requires that permanent displacements be taken into account. A failure to do so affects the values obtained for the structural damping and subsequent displacements. The stiffness is less sensitive to eccentricities. In cases of linear structural response, acceleration time histories represent sufficient data for the application of system identification techniques. A lack of knowledge of the exact behavior of the damping during strucrural yield makes it impossible to compensate for the absence of displacement data when nonlinear effects occur.

The one degree of freedom equation of motion is applicable to partitioned single story frames even after separation has occurred between frame and partitions if the frame contributes the dominant part of the structural stiffness.

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Fig. 2b Bilinear Model.

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Fig. 4. Actual and Simulated Response of the Structure of Fig. 1-b, Fig. 5.









Fig. 8. Experimental Results for the Structure of Fig. 5 ( 9-15 sec.)